Dromion Perturbation for the Davey-Stewartson-1 Equations

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Abstract

The perturbation of the dromion of the Davey-Stewartson-1 equation is studied over the large time.

1 Introduction

In this work we construct the asymptotic solution of the perturbed Davey-Stewartson-1 equations (DS-1):

$$i\partial_t Q + \frac{1}{2}(\partial_\xi^2 + \partial_\eta^2)Q + (G_1 + G_2)Q = \varepsilon iF,$$
$$\partial_\xi G_1 = -\frac{\sigma}{2}\partial_\eta |Q|^2, \quad \partial_\eta G_2 = -\frac{\sigma}{2}\partial_\xi |Q|^2. \tag{1.1}$$

Here ε is small positive parameter, $\sigma = \pm 1$ correspond to so called focusing or defocusing DS-1 equations.

The equations (1.1) describe the interaction between long and short waves on the liquid surface in case the capillary effects and potential flow are taken into account [1, 2]. The existence theorems for the solutions of this equations in the various functional classes are well-known [3, 4]. The inverse scattering transform method for the DS equations was formulated in [5]–[8]. This method allows to construct solitons [9] and to study global properties of the generic solutions, for instance, the asymptotic behaviour over large times [10, 11]. The asymptotics for nonintegrable DS equations were studied in [12].

The perturbations of the equations (1.1) arise due to a small irregularity of bottom or by taking into account the next corrections in more realistic models for the liquid surface considered in [1, 2]. For the first case the perturbation takes the form: $F \equiv AQ$. Here A is real constant and its sign corresponds to decreasing or increasing depth with respect to spatial variable ξ.

We start with the solution of DS-1 constructed in [9]:

$$q(\xi, \eta, t; \rho) = \frac{\rho\lambda\mu \exp(it(\lambda^2 + \mu^2))}{2\cosh(\mu\xi)\cosh(\lambda\eta)(1 - \sigma|\rho|^2\frac{\mu\lambda}{16}(1 + \tanh(\lambda\eta))(1 + \tanh(\mu\xi)))}, \quad (1.2)$$

where λ, μ are positive constants defined by boundary conditions as $\eta \to -\infty$ and $\xi \to -\infty$; ρ is free complex parameter.

This solution decreases with respect to spatial variables exponentially. Besides if $\sigma = 1$ and $\frac{\mu\lambda}{4}|\rho|^2 > 1$ this solution has singularities at some lines $\eta = \text{const}$ and $\xi = \text{const}$.

The solution (1.2) was called dromion in work [7]. The inverse scattering method for dromion-like solution of (1.1) was developed in [7]. From the results of [13], [14] and [10] it follows that the soliton of the DS-2 equations is unstable with respect to small perturbation of the initial data. These results stimulate the studies of the dromion perturbation.

To construct the asymptotic solution over the large times we suppose the parameter ρ depending on a slow time variable $\tau = \varepsilon t$ and obtain an explicit formula for the $\rho(\tau)$.

The possibility of the full investigation of the linearized DS-1 equations plays the main role to construct the perturbation theory of the nonlinear DS-1 equations. For (1+1)-models it has been found in [15], that spectral functions of the Lax pair form the basis of solutions of the linearized nonlinear PDEs. The same idea is true for the DS-1 and DS-2 equations, as shown in [16], [17] and it is used below.

The asymptotic analysis given in this paper is valid for the solutions (1.2) without the singularities. It means if $\sigma = 1$, then $\frac{\mu\lambda}{4}|\rho(\tau)|^2 < 1$. If the coefficient of the perturbation $A > 0$, then $|\rho|$ increases with respect to slow time. It allows to say that the singularity may appear in the leading term of the asymptotics as $\tau \to \infty$. However we can't say this strongly for perturbed dromion in our situation, because our asymptotics is usable only when $\tau \ll \log(\log(\varepsilon^{-1}))$. In general case the appearance of the singularities in the solution of nonintegrable cases of the Davey-Stewartson equations is known phenomenon [18].

The obtained result for the problem about the interaction between the long and short waves on the liquid surface shows when the depth decreases the formal asymptotic solution can be described by adiabatic perturbation theory of the dromion for the focusing DS-1 equations at least for $|t| \ll \varepsilon^{-1}\log(\log(\varepsilon^{-1}))$.

The contents of the various sections are as follows. Section 2 contains a statement of a problem and a result. Section 3 is a brief treatment of solving of linearized DS-1 equations using the basis formed by solutions of the Dirac equation. In Section 4 we apply the results of Section 3 and construct the first correction of an asymptotic expansion for the solution of the problem formulated in Section 1. Section 5 is devoted to reducing of a modulation equation for the dromion parameter. In appendix we demonstrate explicit formulas for functions which are used to obtain the basis set solving of the linearized DS-1 equations.

2   Problem and result

We construct the asymptotic solution of equations (1.1) on $\text{mod}(O(\varepsilon^2))$ uniformly over large t. The perturbation operator is $F \equiv AQ$ and the boundary conditions for G_1 and G_2 are:

$$G_1|_{\xi\to-\infty} = u_1 \equiv \frac{\lambda^2}{2\cosh^2(\lambda\eta)}, \quad G_2|_{\eta\to-\infty} = u_2 \equiv \frac{\mu^2}{2\cosh^2(\mu\xi)}. \quad (2.1)$$

We seek the asymptotic solution as a sum of two first terms of asymptotic expansions:

$$Q(\xi, \eta, t, \varepsilon) = W(\xi, \eta, t, \tau) + \varepsilon U(\xi, \eta, t, \tau),$$
$$G_1(\xi, \eta, t, \varepsilon) = g_1(\xi, \eta, t, \tau) + \varepsilon V_1(\xi, \eta, t, \tau), \qquad (2.2)$$
$$G_2(\xi, \eta, t, \varepsilon) = g_2(\xi, \eta, t, \tau) + \varepsilon V_2(\xi, \eta, t, \tau),$$

where $\tau = \varepsilon t$ is slow time. The leading term of the asymptotics has the form:

$$W(\xi, \eta, t, \tau) = q(\xi, \eta, t; \rho(\tau)),$$

and g_1, g_2 are:

$$g_1(\xi, \eta, t, \tau) = u_1 - \frac{\sigma}{2} \int_{-\infty}^{\xi} d\xi' \partial_\eta |W(\xi', \eta, t, \tau)|^2,$$

$$g_2(\xi, \eta, t, \tau) = u_2 - \frac{\sigma}{2} \int_{-\infty}^{\eta} d\eta' \partial_\xi |W(\xi, \eta', t, \tau)|^2.$$

Denote by $\gamma(\tau) = 1 - \sigma \frac{\mu \lambda}{4} |\rho(\tau)|^2$ and $\gamma_0 = \gamma(0)$. The final result is given by

Theorem 1. *If*

$$\gamma(\tau) = \gamma_0^{\exp(2A\tau)}, \quad Arg(\rho(\tau)) \equiv const,$$

where $\gamma_0 > 1$ at $\sigma = -1$ and $0 < \gamma_0 < 1$ at $\sigma = 1$, then the asymptotic solution (2.2) with respect to $mod(O(\varepsilon^2))$ is useful uniformly over $t = O(\varepsilon^{-1})$.

Remark 1. When the time is larger than ε^{-1}, namely, $t \ll \varepsilon^{-1} \log(\log(\varepsilon^{-1}))$, the formulas (2.2) are asymptotic solution of (1.1) with respect to $mod(o(1))$ only.

Remark 2. D. Pelinovsky notes to author, that the modulation of the parameter $|\rho|$ may be obtained out of the "energetic equality" [19]:

$$\partial_t \int \int_{\mathbb{R}^2} d\xi d\eta |Q|^2 = \varepsilon \int \int_{\mathbb{R}^2} d\xi d\eta (Q\bar{F} - \bar{Q}F).$$

3 Solution of linearized equations

In the present section we obtain the formulas for solution of the linearized DS-1 equations on the dromion as a background:

$$i\partial_t U + (\partial_\xi^2 + \partial_\eta^2)U + (G_1 + G_2)U + (V_1 + V_2)Q = iF$$
$$\partial_\xi V_1 = -\frac{\sigma}{2}\partial_\eta(Q\bar{U} + \bar{Q}U), \quad \partial_\eta V_2 = \frac{-\sigma}{2}\partial_\xi(Q\bar{U} + \bar{Q}U). \qquad (3.1)$$

The results of inverse scattering transform [7] and the set of the basic functions [16] are used for solving of the linearized equations by Fourier method. It is reminded in Subsection 3.1.

However in contrast to the work [16] here the solution of the DS-1 equations with nonzero boundary conditions is considered. It leads to changing of the dependency of the scattering data with respect to time (see also [7]) and of the formulas which define the dependency of Fourier coefficients of the solution for the linearized DS-1 equations in contrast to obtained in [16]. This is explained in Subsection 3.2.

3.1   Basic set for solving of the linearized DS-1 equation

In the inverse scattering transform one use the matrix solution of the Dirac system to solve the DS-1 equations (see [5]-[7]):

$$\begin{pmatrix} \partial_\xi & 0 \\ 0 & \partial_\eta \end{pmatrix}\psi = -\frac{1}{2}\begin{pmatrix} 0 & Q \\ \sigma\bar{Q} & 0 \end{pmatrix}\psi. \tag{3.2}$$

Let ψ^+ and ψ^- be the matrix solutions of the Goursat problem for the Dirac system with the boundary conditions (see [7]):

$$\begin{aligned}
&\psi_{11}^+|_{\xi\to-\infty} = \exp(ik\eta), && \psi_{12}^+|_{\xi\to-\infty} = 0, \\
&\psi_{21}^+|_{\eta\to\infty} = 0, && \psi_{22}^+|_{\eta\to-\infty} = \exp(-ik\xi); \\
&\psi_{11}^-|_{\xi\to-\infty} = \exp(ik\eta), && \psi_{12}^-|_{\xi\to\infty} = 0, \\
&\psi_{21}^-|_{\eta\to-\infty} = 0, && \psi_{22}^-|_{\eta\to-\infty} = \exp(-ik\xi).
\end{aligned} \tag{3.3}$$

Denote by $\psi_{(j)}^+$, $j = 1, 2$, the columns of the matrix ψ^+. The column $\psi_{(1)}^+$ is the solution of two systems of equations. An additional system of time evolution is

$$\partial_t\psi_{(1)}^+ = ik^2\psi_{(1)}^+ + i\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}(\partial_\xi - \partial_\eta)^2\psi_{(1)}^+$$

$$+ i\begin{pmatrix} 0 & Q \\ \sigma\bar{Q} & 0 \end{pmatrix}(\partial_\xi - \partial_\eta)\psi_{(1)}^+ + \begin{pmatrix} iG_1 & -i\partial_\eta Q \\ i\sigma\partial_\xi\bar{Q} & -iG_2 \end{pmatrix}\psi_{(1)}^+. \tag{3.4}$$

One can obtain the equation like this for the other columns of the matrices $\psi^\pm$. These equations are differ from (3.4) by the sign of ik^2 in first term of the right hand side only.

Below we write two bilinear forms defining analogues of the direct and inverse Fourier transforms. First bilinear form is

$$(\chi, \mu)_f = \int_{-\infty}^\infty \int_{-\infty}^\infty d\xi d\eta(\chi_1\mu_1\,\sigma\bar{f} + \chi_2\mu_2\,f). \tag{3.5}$$

Here χ_i and μ_i are the elements of the columns χ and μ.

Denote by $\phi_{(1)}$ and $\phi_{(2)}$ the solutions conjugated to $\psi_{(1)}^+$ and $\psi_{(2)}^-$ with respect to the bilinear form (3.5).

Using the formulas for the scattering data ([7]) one can write these data as:

$$s_1(k, l) = \frac{1}{4\pi}(\psi_{(1)}^+(\xi, \eta, k), E_{(1)}(il\xi))_Q, \tag{3.6}$$

$$s_2(k, l) = \frac{1}{4\pi}(\psi_{(2)}^-(\xi, \eta, k), E_{(2)}(il\eta))_Q. \tag{3.7}$$

Here $E(z) = \mathrm{diag}(\exp(z), \exp(-z))$.

It is shown ([7]) the elements of the matrices $\psi^\pm$ are analytic functions with respect to the variable k when $\pm Im(k) > 0$. Using the scattering data one can write the nonlocal Riemann-Hilbert problem for the ψ_{11}^- and ψ_{12}^+ on the real axes ([7]):

$$\psi_{11}^-(\xi, \eta, k) = \exp(ik\eta) + \exp(ik\eta)\left(\exp(-ik\eta)\int_{-\infty}^\infty dl\, s_1(k, l)\psi_{12}^+(\xi, \eta, l) \right)^-,$$

$$\psi_{12}^+(\xi, \eta, k) = \exp(-ik\xi)\left(\exp(ik\xi)\int_{-\infty}^\infty dl\, s_2(k, l)\psi_{11}^-(\xi, \eta, l) \right)^+.$$

Here

$$\left(f(k) \right)^{\pm} = \frac{1}{2i\pi} \int_{-\infty}^{\infty} \frac{dk'\, f(k')}{k' - (k \pm i0)}.$$

The Riemann-Hilbert problem for ψ_{21}^{-} and ψ_{22}^{+} has the form:

$$\psi_{21}^{-}(\xi, \eta, k) = \exp(ik\eta) \left(\exp(-ik\eta) \int_{-\infty}^{\infty} dl\, s_1(k,l)\psi_{22}^{+}(\xi, \eta, l) \right)^{-},$$

$$\psi_{22}^{+}(\xi, \eta, k) = \exp(-ik\xi) + \exp(-ik\xi) \left(\exp(ik\xi) \int_{-\infty}^{\infty} dl\, s_2(k,l)\psi_{21}^{-}(\xi, \eta, l) \right)^{+}.$$

Introduce second bilinear form

$$\langle \chi, \mu \rangle_s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk dl (\chi^1(l)\mu^1(k)\, s_2(k,l) + \chi^2(l)\mu^2(k)\, s_1(k,l)), \qquad (3.8)$$

where χ^j is the element of the row χ.

Denote by $\varphi^{(j)}$, $j = 1, 2$, the row conjugated to $\psi^{(j)} = [\psi_{j1}^{-}, \psi_{j2}^{+}]$ with respect to the bilinear form (3.8). Formulate the result about the decomposition obtained in [16].

Theorem 2. *Let Q be such that $\partial^{\alpha} Q \in L_1 \cap C^1$ for $|\alpha| \leq 3$, if a function f is such that $\partial^{\alpha} f(\xi, \eta) \in L_1 \cap C^1$ for $|\alpha| \leq 4$, then one can represent the f in the form*

$$f = \frac{-1}{\pi} \langle \psi^{(1)}(\xi, \eta, l), \varphi^{(1)}(\xi, \eta, k) \rangle_{\hat{f}}, \qquad (3.9)$$

where

$$\hat{f} = \frac{1}{4\pi} (\psi_{(1)}^{+}(\xi, \eta, k), \phi_{(1)}(\xi, \eta, l))_f. \qquad (3.10)$$

3.2   Solving the linearized equation using Fourier method

In the preceding subsection we have state that the bilinear forms (3.5) and (3.8) may be used to decomposition like as a Fourier integrals. The present subsection shows how to use this decomposition for solving of linearized DS-1 equation.

Theorem 3. *Let Q be the solution of the DS-1 equations with the boundary conditions $G_1|_{\xi \to -\infty} = u_1$ and $G_2|_{\eta \to -\infty} = u_2$, and Q satisfies the conditions of Theorem 2, the solution of the first of the linearized DS-1 equation is smooth and integrable function U with respect to ξ and η, where $\partial^{\alpha} U \in L_1 \cap C^1$ and $\partial^{\alpha} F \in L_1 \cap C^1$, for $|\alpha| \leq 4$ and $t \in [0, T_0]$. Then*

$$\partial_t \hat{U} = i(k^2 + l^2)\hat{U} + \int_{-\infty}^{\infty} dk' \hat{U}(k - k', l, t)\chi(k') + \int_{-\infty}^{\infty} dl' \hat{U}(k, l - l', t)\kappa(l') + \hat{F}.$$

$$(3.11)$$

If the boundary conditions for the solution of the DS-1 equation equal to zero, then $\chi \equiv \kappa \equiv 0$. In this case the formulas of the Theorem 3 allow to solve the linearized DS-1 equation in the explicit form. It was be done in [16]. In contrast of [16], we consider

here the solution of the DS-1 equation with nonzero boundary conditions. It leads to integral terms in the formula (3.11). In order to solve the linearized DS-1 equation we must transform the formula (3.11). In the right hand side of (3.11) the integral terms are the convolutions. Go over to the equations for the Fourier transform of the functions $\hat{U}(k, l, t, \tau)$ with respect to variables k and l. As a result we obtain the linear Schrödinger equation:

$$i\partial_t \tilde{U} + (\partial_\xi^2 + \partial_\eta^2)\tilde{U} + (u_2(\xi\mu) + u_1(\eta\lambda))\tilde{U} = \tilde{F}. \tag{3.12}$$

Here

$$\tilde{U}(\xi, \eta, t, \tau) = \frac{1}{2\pi}\int_{\mathbb{R}^2} dk dl \hat{U}(k, l, t, \tau)\exp(-ik\eta - il\xi);$$

$$\tilde{F}(\xi, \eta, t, \tau) = \frac{1}{2\pi}\int_{\mathbb{R}^2} dk dl \hat{F}(k, l, t, \tau)\exp(-ik\eta - il\xi). \tag{3.13}$$

The same equation without the right hand side (as $\tilde{F} \equiv 0$) was obtained in [7] for the time evolution of the scattering data for the DS-1 equation.

One can construct the solution of the Cauchy problem for the equations (3.12) with the initial condition $\tilde{U}(\xi, \eta) = 0$ by the separation of the variables. In our case the solution of the equations (3.12) obtained by the Fourier method has the form:

$$\tilde{U}(\xi, \eta, t) = \frac{1}{2\pi}\int_{\mathbb{R}^2} dm dn \breve{U}(m, n, t)X(\xi, m)Y(\eta, n)\exp(-it(m^2 + n^2))$$

$$+ \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} dn \breve{U}_\mu(n, t)Y(\eta, n)X_\mu(\xi)\exp(-it(n^2 - \mu^2))$$

$$+ \frac{1}{\sqrt{2\pi}}\int_{\mathbb{R}} dm \breve{U}_\lambda(m, t)X(\xi, m)Y_\lambda(\eta)\exp(-it(m^2 - \lambda^2))$$

$$+ \breve{U}_{\mu,\lambda}X_\mu(\xi)Y_\lambda(\eta)\exp(it(\mu^2 + \lambda^2)). \tag{3.14}$$

Here we use notations:

$$X(m, \xi) = \frac{\mu\tanh(\mu\xi) + im}{im - \mu}\exp(-im\xi), \quad X_\mu(\xi) = \frac{1}{2\cosh(\mu\xi)};$$

$$Y(n, \eta) = \frac{\lambda\tanh(\lambda\eta) + in}{in - \lambda}\exp(-in\eta), \quad Y_\lambda(\eta) = \frac{1}{2\cosh(\lambda\eta)};$$

$$\partial_t\breve{U} = i(m^2 + n^2)\breve{U} + \breve{F}(m, n, t), \quad \partial_t\breve{U}_\mu = i(n^2 - \mu^2)\breve{U} + \breve{F}_\mu(n, t),$$

$$\partial_t\breve{U}_\lambda = i(m^2 - \lambda^2)\breve{U} + \breve{F}_\lambda(m, t), \quad \partial_t\breve{U}_{\mu\lambda} = -i(\lambda^2 + \mu^2)\breve{U} + \breve{F}_{\mu\lambda}(t), \tag{3.15}$$

$$\breve{U}|_{t=0} = \breve{U}_\mu|_{t=0} = \breve{U}_\lambda|_{t=0} = \breve{U}_{\mu\lambda}|_{t=0} = 0;$$

$$\breve{F}(m, n, t) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t)\bar{X}(m, \xi)\bar{Y}(n, \eta),$$

$$\breve{F}_\mu(n, t) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t)\bar{X}_\mu(\xi)\bar{Y}(n, \eta),$$

$$\breve{F}_\lambda(m, t, \tau) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t)\bar{X}(m, \xi)\bar{Y}_\lambda(\eta),$$

$$\breve{F}_{\mu\lambda}(t, \tau) = \int_{\mathbb{R}^2} d\xi d\eta \tilde{F}(\xi, \eta, t)\bar{X}_\mu(\xi)\bar{Y}_\lambda(\eta). \tag{3.16}$$

To solve the linearizing equations we must apply the formulas (3.10), (3.13), (3.16), solve the Cauchy problems (3.15) and apply formulas (3.14), inverse Fourier transform to $\tilde{U}$ and then apply formula (3.9) for $\hat{U}$. As a result we obtain the solution of the linearized DS-1 equations. The functions $V_{1,2}$ may be obtained by direct integrating of second and third equations from (3.1).

4   Equation for the first correction

This and next section contain a proof of the Theorem 1. In this part the equation for the slow modulation of the parameter $\rho(\tau)$ is obtained. This equation is necessary and sufficient condition for the uniform boundedness of the first correction of the expansion (2.2) over $t = O(\varepsilon^{-1})$.

Some complication appears when one use the Fourier method from preceding section to solve the linearized DS-1 equations on dromion as a background with slow varying parameter $\rho(\tau)$. In this case the basic functions used for solving the linearized equations depend on slow variable τ also and they are the basic set of asymptotic solutions with respect to $\mathrm{mod}(O(\varepsilon))$ only.

Substitute the formula (2.2) into the equations (1.1). Equate the coefficients with the same power of ε. The equations as ε^0 are realized since W, g_1, g_2 are the asymptotic solution of nonperturbed DS-1 equations. For the first correction we obtain the linearized DS-1 equations:

$$
\begin{aligned}
&i\partial_t U + (\partial_\xi^2 + \partial_\eta^2)U + (g_1 + g_2)U + (V_1 + V_2)W = iH \\
&\partial_\xi V_1 = -\frac{\sigma}{2}\partial_\eta(W\bar{U} + \bar{W}U), \quad \partial_\eta V_2 = \frac{-\sigma}{2}\partial_\xi(W\bar{U} + \bar{W}U),
\end{aligned}
\tag{4.1}
$$

where

$$
H = AW - \partial_\tau W.
$$

Before to use the formulas from the preceding section we reduce the form of the right hand side in first of the equations (4.1). In the leading term the parameter ρ depends on the τ only. The other parameters of the dromion (λ and μ) depend only on the boundary conditions and have not changes under perturbation of the equations. For more convenience we represent $\rho(\tau) = r(\tau)\exp(i\alpha(\tau))$ where $r(\tau) = |\rho(\tau)|$ and $\alpha(t) = \mathrm{Arg}\rho(\tau)$. The derivation of W with respect to slow variable τ can be written as:

$$
\partial_\tau W = \partial_r W r' + \partial_\alpha W \alpha'.
$$

Here the derivatives r' and α' are unknown yet.

Compute the function $\hat{H}$. Using the Theorem 2 and formulas for the functions ψ_+ and ϕ (see Appendix) we obtain:

$$
\widehat{H}(k,l,t,\tau) = \exp(-it(\lambda^2 + \mu^2))\left(\widehat{P}(k,l;\rho) - \widehat{R}(k,l;\rho)\partial_\tau\rho\right),
$$

where

$$
\widehat{P}(k,l;\rho) = \exp(it(\lambda^2 + \mu^2))\widehat{AW}, \quad \widehat{R}(k,l;\rho) = \exp(it(\lambda^2 + \mu^2))\widehat{\partial_\tau W}.
$$

In these formulas we have write the dependence on time in explicit form. It allows to remove the secular terms (increasing with respect to t) from the asymptotic solution (2.2) using modulation of the parameters $r(\tau)$ and $\alpha(\tau)$.

The differential equations for $\breve{U}$ have the forms:

$$\partial_t \breve{U} = i(m^2 + n^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}(m,n;\rho) - \breve{R}(m,n;\rho)),$$

$$\partial_t \breve{U}_\mu = i(n^2 - \mu^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_\mu(n;\rho) - \breve{R}_\mu(n;\rho)),$$

$$\partial_t \breve{U}_\lambda = i(m^2 - \lambda^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_\lambda(m;\rho) - \breve{R}_\lambda(m;\rho)),$$

$$\partial_t \breve{U}_{\mu\lambda} = -i(\lambda^2 + \mu^2)\breve{U} + \exp(-it(\lambda^2 + \mu^2))(\breve{P}_{\mu\lambda}(\rho) - \breve{R}_{\mu\lambda}(\rho)).$$

Solutions of these equations are

$$\breve{U}(m,n,t) = \frac{\mu^2 + \lambda^2}{n^2 + m^2 + \mu^2 + \lambda^2}\left(\breve{P}(m,n;\rho) - \breve{R}(m,n;\rho)\right)\exp(-it(\mu^2 + \lambda^2)),$$

$$\breve{U}_\lambda(m,t) = \frac{\mu^2 + \lambda^2}{m^2 + \mu^2}\left(\breve{P}_\lambda(m;\rho) - \breve{R}_\lambda(m;\rho)\right)\exp(-it(\mu^2 + \lambda^2)),$$

$$\breve{U}_\mu(n,t) = \frac{\mu^2 + \lambda^2}{n^2 + \lambda^2}\left(\breve{P}_\mu(n;\rho) - \breve{R}_\mu(n;\rho)\right)\exp(-it(\mu^2 + \lambda^2)),$$

$$\breve{U}_{\mu\lambda}(t) = \left(\breve{P}_{\mu\lambda}(m;\rho) - \breve{R}_{\mu\lambda}(m;\rho)\right)t\exp(-it(\mu^2 + \lambda^2)).$$

One can see that the secular terms may appear because of the last term in the equation for $\breve{U}_{\mu\lambda}$. The eliminating of this term leads us to the equation for $\rho(\tau)$:

$$\breve{R}_{\mu\lambda} - \breve{P}_{\mu\lambda} = 0, \quad \rho|_{\tau=0} = \rho_0. \tag{4.2}$$

As a result $\breve{U}_{\mu\lambda} \equiv 0$. The another solutions $\breve{U}(m,n,t)$, $\breve{U}_\mu(n,t)$ and $\breve{U}_\lambda(m,t)$ are bounded with respect to all arguments and over all times.

One must return to the original of the images $\breve{U}$ to say about boundedness of the solution U, V_1, V_2 for the equations (4.1). One can see the direct (from U into $\breve{U}$) and inverse (from $\breve{U}$ into U) integral transforms as the Fourier transform from the smooth and exponentially decreasing functions with respect to the corresponding variables. The Fourier transform moves such functions into analytic functions near the real axis. The inverse transform moves these analytic functions into the exponential decreasing functions. So the solution of (4.1) is bounded and decreasing exponentially with respect to the spatial variables.

5   Modulation equation

Here the equation (4.2) for the parameter $\rho(\tau)$ is reduced to the more convient form. Write the derivative of the leading term with respect to the slow variable τ.

$$i\partial_\tau W = -\alpha' W - iW\frac{r'}{r} + \frac{2iW}{1 - \sigma r^2\frac{\mu\lambda}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta))}\frac{r'}{r}.$$

Denote by $\Gamma = \frac{\mu\lambda}{4}r^2$ and compute the images $(\breve{\cdot})_{\mu\lambda}$ of every term.

$$(\breve{W})_{\mu\lambda} = \frac{\sigma\bar{\rho}\exp(-it(\lambda^2 + \mu^2))}{8}(\sigma\Gamma - 1)\log|1 - \sigma\Gamma|;$$

$$(i\breve{W})_{\mu\lambda} = -i\sigma\Gamma\frac{\rho\exp(-it(\lambda^2 + \mu^2))}{8}.$$

Denote the image of the last term by $\breve{h}_{\mu\lambda}$. Its image has the form:

$$\breve{h}_{\mu\lambda} = \frac{r'}{r}\frac{\sigma\bar{\rho}\exp(-it(\lambda^2 + \mu^2))}{8}(1 - \sigma\Gamma)\left(\frac{1}{1 - \sigma\Gamma} - 1 - \log|1 - \sigma\Gamma|\right).$$

The image $(\breve{\cdot})$ of AW has the similar form.

Substitute these formulas into (4.2) and separate the real and the imaginary parts of this equation, then

$$\alpha' = 0,$$

$$\frac{r'}{r}(\sigma\Gamma - 1)\log|1 - \sigma\Gamma| - \frac{r'}{r}\left(\sigma\Gamma - (1 - \sigma\Gamma)\log|1 - \sigma\Gamma|\right)$$

$$+ A(\sigma\Gamma - 1)\log|1 - \sigma\Gamma| = 0.$$

Use the notation for Γ, then the second equation has the form:

$$\frac{d\Gamma}{d\tau} = -2\sigma A(1 - \sigma\Gamma)\log|1 - \sigma\Gamma|.$$

This equation defines the evolution of the absolute value of the complex parameter ρ. The argument of this parameter do not change under the perturbation $F = AQ$.

Denote by $\gamma = 1 - \sigma\Gamma$, rewrite the equation for γ. Then we obtain:

$$\gamma' = 2A\gamma\log(\gamma).$$

The solution for this equation has the form:

$$\gamma(\tau) = \exp(C\exp(2A\tau)),$$

where $\gamma|_{\tau=0} = \exp(C)$, then we can write the $\gamma(\tau)$ in the form:

$$\gamma(\tau) = \gamma_0^{\exp(2A\tau)}.$$

The Theorem 1 is proved.

Acknowledgements

Appendix

A.1 Explicit formulas

Here we remain the explicit forms of the solution for the Dirac equation with the dromion-like potential. These forms were obtained in [7]. In our computations we use first column of the matrix ψ^+ only.

$$\begin{pmatrix} \psi_{11}^+ \\ \psi_{21}^+ \end{pmatrix} = \begin{pmatrix} \exp(ik\eta) \\ 0 \end{pmatrix} + \frac{\int_\eta^\infty \frac{dp\lambda\exp(ikp))}{2\cosh(\lambda p)}}{(1 - \sigma|\rho|^2\frac{\lambda\mu}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta)))}$$

$$\times \begin{pmatrix} \frac{-\sigma|\rho|^2\lambda\mu(1+\tanh(\mu\xi))}{8\cosh(\lambda\eta)} \\ \frac{\sigma\bar\rho\mu\exp(-it(\lambda^2+\mu^2))}{2\cosh(\mu\xi)} \end{pmatrix}. \tag{A.1}$$

A.2 Solutions of conjugated equations

Here we write the problems for the functions conjugated to $\psi^\pm$ with respect to the bilinear forms.

The matrix ϕ is the solution of the boundary problem conjugated to the solutions of the problem (3.2), (2.1) with respect to the bilinear form (3.5). First column of the matrix ϕ is the solution of the integral equation:

$$\phi_{11}(\xi,\eta,l,t) = \exp(il\xi) + \frac{1}{2}\int_{-\infty}^\eta d\eta' Q(\xi,\eta',t)\phi_{21}(\xi,\eta',l,t),$$

$$\phi_{21}(\xi,\eta,l,t) = \frac{-1}{2}\int_\xi^\infty d\xi' \bar Q(\xi',\eta,t)\phi_{11}(\xi',\eta,l,t).$$

The explicit formula for first column of the matrix ϕ used in Section 4 for the dromion potential has the form:

$$\begin{pmatrix} \phi_{11} \\ \phi_{21} \end{pmatrix} = \begin{pmatrix} \exp(il\xi) \\ 0 \end{pmatrix} + \frac{\int_\xi^\infty \frac{dp\mu\exp(ilp))}{2\cosh(\mu p)}}{(1 - \sigma|\rho|^2\frac{\lambda\mu}{16}(1 + \tanh(\mu\xi))(1 + \tanh(\lambda\eta)))}$$

$$\times \begin{pmatrix} \frac{-\sigma|\rho|^2\lambda\mu(1+\tanh(\lambda\eta))}{8\cosh(\mu\xi)} \\ \frac{-\sigma\bar\rho\lambda\exp(-it(\lambda^2+\mu^2))}{2\cosh(\lambda\eta)} \end{pmatrix}. \tag{A.2}$$

Second bilinear form (3.8) allows to write the integral equations conjugated to the integral equations which were obtained from the nonlocal Riemann-Hilbert equation for the matrices $\psi^\pm$ in [7]. Using the equation for the matrix $\varphi(\xi,\eta,l)$ one can show that φ_{11} and φ_{21} are the solutions of the boundary problem for the Dirac system or the equivalent integral equations:

$$\varphi_{11}(\xi,\eta,l) = \exp(il\xi) + \frac{1}{2}\int_{-\infty}^\eta d\eta' Q(\xi,\eta',t)\varphi_{21}(\xi,\eta',l),$$

$$\varphi_{21}(\xi,\eta,l) = \frac{1}{2}\int_{-\infty}^\xi d\xi' \bar Q(\xi',\eta,t)\varphi_{11}(\xi',\eta,l).$$

The functions φ_{11} and φ_{21} have the similar form as the functions ϕ_{11} and ϕ_{21}.

References

[1] Davey A. and Stewartson K., On the Three-Dimensional Packets of Surface Waves, *Proc. R. Soc. London Ser. A*, 1974, V.338, 101–110.

[2] Djordjevic V.D. and Redekopp L.G., On Two-Dimensional Packets of Capillary- Gravity Waves, *J. Fluid Mech.*, 1977, V.79, 703–714.

[3] Ghidaglia J.-M. and Saut J.-K., On the Initial Value Problem for the Davey-Stewartson Systems, *Nonlinearity*, 1990, V.3, 475–506.

[4] Shakir'yanov M.M., Solvability of the Initial-Boundary Problem for the Davey-Stewartson Type System of Equations, *in:* Integrability in the Dynamical Systems, *Inst. of Math. Ufa*, 1994, 49–61.

[5] Nizhnik L.P., Inverse Problems for Hyperbolic Equations, Naukova Dumka, Kiev, 1991.

[6] Fokas A.S. and Ablowitz M.J. On the Inverse Scattering Transform of Multidimensional Nonlinear Equations Related to First-Order Systems in the Plane, *J. Math. Phys.*, 1984, V.25, 2494–2505.

[7] Fokas A.S. and Santini P.M., Dromions and a Boundary Value Problem For the Davey-Stewartson I Equation, *Physica D*, 1990, V.44, 99–130.

[8] Fokas A.S. and Sung L.J., On the Solvability on the N-Wave, Davey-Stewartson and Kadomtsev-Petviashvili Equations. *Inv. probl.*, 1992, V.8, 673–708.

[9] Boiti M., Leon J., Martina L. and Pempinelli F., Scattering of Localized Solitons in the Plane, *Phys. Lett. A*, 1988, V.132, 432–439.

[10] Kiselev O.M., Asymptotic Behavior of Solitonless Solution of the Davey-Stewartson-2 Equation, *Differentsial'nye Uravneniya*, 1997, V.33, 812–819.

[11] Kiselev O.M., An Asymptotic Solution of the Cauchy Problem for the Davey-Stewartson-1 Equation, *Teor. Matem. Fiz.*, 1998, V.114, 104–114.

[12] Nakao H. and Hirata H., Global Existence and Asymptotic Behavior in Time of Small Solutions to the Elliptic-Hyperbolic Davey-Stewartson System, *Nonlinearity*, 1996, V.9, 1387–1409.

[13] Gadyl'shyn R.R. and Kiselev O.M., On Nonsoliton Structure of Scattering Data under Perturbation of Two-Dimensional Soliton for Davey-Stewartson Equation II, *Teor. Matem. Fiz.*, 1996, V.106, 167–173.

[14] Gadylshin R.R. and Kiselev O.M., Structural Instability of Soliton for Davey-Stewartson II Equation, *Teor. Matem. Fiz.*, 1999, V.118, 354–361.

[15] Kaup D.J., Closure of the Squared Zakharov-Shabat Eigenstates, *J. of Math. Anal. and Appl.*, 1976, V.54, 849–864.

[16] Kiselev O.M., Perturbation Theory for the Dirac Equation in Two-Dimensional Space, *J. Math. Phys.*, 1998, V.39, 2333–2345.

[17] Kiselev O.M., Basic Functions Associated With Two Dimensional Dirac System, *Func. Anal. Appl.*, 1998, V.32, 56–59.

[18] Papanicolaou G.S., Sulem C., Sulem P.L. and Wang X.P., The Focusing Singularity of the Davey-Stewartson Equations for Gravity-Capillary Surface Waves, *Physica D*, 1994, V.72, 61–86.

[19] Pelinovsky D., Private Communication, 1999.